Optical Signal Processing

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Chapter 5
Imaging and Diffraction-Limited Imaging

Introduction

- **Imaging is about reproducing the field, or more often the irradiance pattern of an object or scene, at an image plane.**
- Geometrical optics, where optical rays are assumed to travel in rectilinear fashion without diffraction, is used extensively in lens and optical system design.
- A proficient approach for image modeling draws on both geometrical optics and diffraction theory.

Geometrical Imaging Concepts

- In order to form a real image, light from an arbitrary object point must be collected and focused at the image plane.
- For the imaging situation, **the lens law describes the relationship needed under the paraxial condition (small ray angles relative to the optical axis) for "best focus" imaging**:

$$\frac{1}{z_1}+\frac{1}{z_2}=\frac{1}{f}$$

- *f* is the lens focal length, z_1 is the distance from the object to the *front principal plane* of the lens, and z_2 is the distance from the *back principal plane* to the image location.

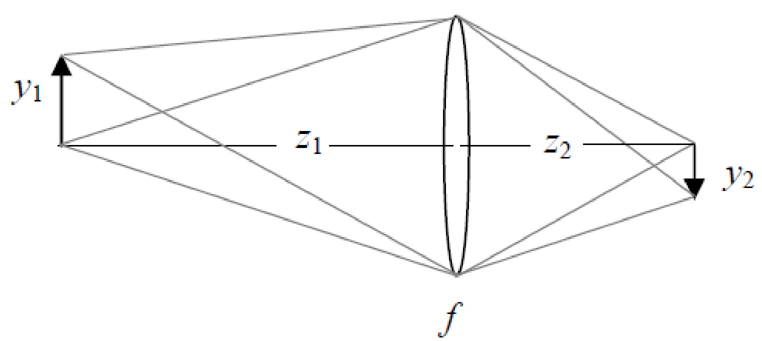


Fig. 1. Geometrical imaging with a thin positive lens of focal length *f*.

Geometrical Imaging Concepts

- To form a real image, z_1 and z_2 are positive and the lens focal length f is positive.
- A "positive" lens (positive-valued f) converges light rays, whereas a "negative" lens (negative-valued f) diverges rays.
- **Practical imaging systems use combinations of lenses to control aberrations, but imaging still requires a positive focal length for the combined lens group.**

Geometrical Imaging Concepts

- The ratio of the image height y_2 to the object height y_1 is known as the *transverse magnification* M_t , which for a single lens system is given by:

$$M_t = \frac{y_2}{y_1} = -\frac{z_2}{z_1}.$$

- The minus sign indicates an inverted image.
- **An imaging system is characterized by its pupils.**
- Pupils are virtual apertures that indicate the "opening" available to collect light from the object (entrance pupil) and the "opening" from which the collected light exits on its way to form an image (exit pupil).

Geometrical Imaging Concepts

- The pupils are images of the physical element in the system, known as the aperture stop, which limits the collection of light.
- The lens is the stop for the system in Fig. 1.
- The stop generates the fundamental diffractive effects in the image: **the diffractive effects due to the stop represent the fundamental performance limit of an imaging system.**

Geometrical Imaging Concepts

- **<u>Fig. 1-1 illustrates that the physical elements of a system (lenses, mirrors, iris, etc.) can be reduced to entrance pupil (EP) and exit pupil (XP) models.</u>**
- The distance from an object point on the optical axis to the EP is z_{EP} and the distance from the XP to the axial image point is z_{XP}.
- The entrance pupil diameter is D_{EP} and the exit pupil diameter is D_{XP}.

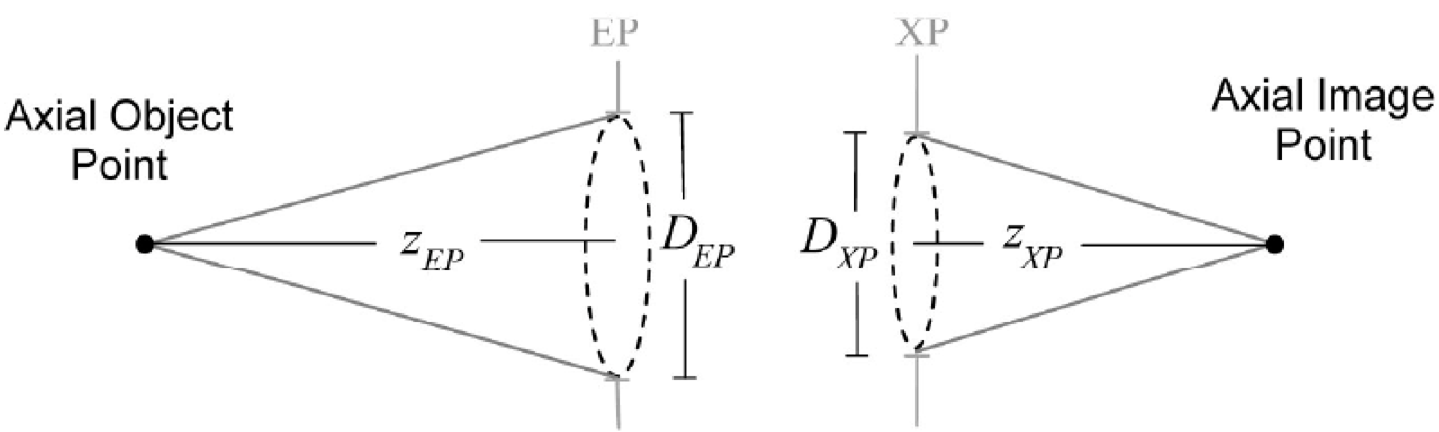


Fig. 1-1. Entrance pupil (EP) and exit pupil (XP) model of an imaging system.

Geometrical Imaging Concepts

- The important parameter of an imaging system is the *f*-number (*f*/#): the most useful form is the paraxial working *f*/# given by

$$f/\# = \frac{z_{XP}}{D_{XP}}$$

- **The system aperture stop leads to the fundamental diffractive effects in the image.**
- For a thin lens imaging system, $z_1 = z_{EP}$, $z_2 = z_{XP}$, and $D_{EP} = D_{XP}$ = lens diameter.

Coherent Imaging Theory

- **<u>Fig.1-2 shows the general imaging arrangement.</u>**

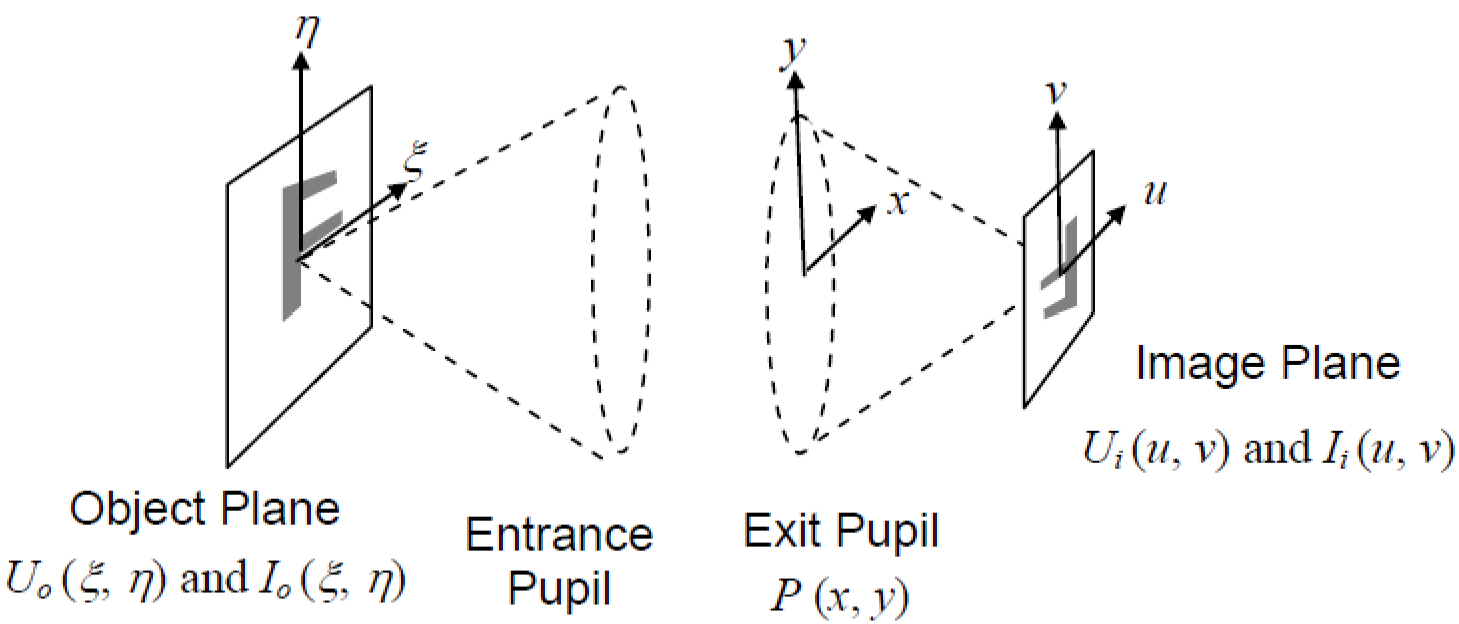


Fig. 1-2. Imaging simulation coordinate definitions.

Coherent Imaging Theory

- **<u>Imaging with coherent illumination, such as with a coherent laser, can be described with a convolution operation involving the optical field.</u>**
- The process is expressed by:

$$U_i(u,v) = h(u,v) \otimes U_g(u,v), \quad (1)$$

It should be noted that we are only interested in the intensity distribution in the image plane.

- where *u, v* are the image plane spatial coordinates, U_i is the field at the image plane, *h* is the impulse response for the imaging system, and **<u>U_g is the ideal geometrical-optics predicted image field, which is a scaled copy of the object field $U_o(x, y)$:</u>**

$$U_g(u,v) = \frac{1}{|M_t|} U_o\left(\frac{u}{M_t}, \frac{v}{M_t}\right).$$

- Note that if M_t is negative, then the resulting image will appear inverted relative to the object.

Coherent Imaging Theory

- In Eq. (1) the ideal geometrical field is "blurred" through the convolution with the impulse function.
- In the frequency domain the corresponding spectra for Eq. (1) are related by:

$$G_i(f_U, f_V) = H(f_U, f_V) G_g(f_U, f_V), \qquad (2)$$

- **<u>where *H* is the coherent image transfer function (or amplitude transfer function) and is defined as:</u>**

$$H(f_U, f_V) = P(-\lambda z_{XP} f_U, -\lambda z_{XP} f_V),$$

Refer to Eq. (5-40) in Fourier Optics [two times FT of $P(\lambda z_{XP} f_U, \lambda z_{XP} f_V)$ -> inverted].

- where *P* is the pupil function of the system.

It should be noted that we are only interested in the intensity distribution in the image plane.

- The negative signs in the pupil arguments give a scaled, inverted pupil function.
- **<u>In the system with a perfect pupil function, only the boundaries of the pupil are involved with the diffractive effects (diffraction limited).</u>**

Coherent Transfer Function Examples

- For the first example, a square pupil function is:

$$P(x,y) = \mathrm{rect}\left(\frac{x}{2w_{XP}}\right)\mathrm{rect}\left(\frac{y}{2w_{XP}}\right). \quad (3)$$

- From Eq. (3) the coherent transfer function is:

$$H(f_U, f_V) = \mathrm{rect}\left(\frac{-\lambda z_{XP} f_U}{2w_{XP}}\right)\mathrm{rect}\left(\frac{-\lambda z_{XP} f_V}{2w_{XP}}\right). \quad (4)$$

- Since the rectangle function is symmetric, the negative signs can be ignored.
- **The coherent cutoff frequency along the *u* or *v* direction is defined as** (5)

$$f_0 = \frac{w_{XP}}{\lambda z_{XP}}.$$

- **Spatial frequencies with values greater than f_0 will not be preserved in the image plane field.**

Coherent Transfer Function Examples

- A second example is the circular pupil function:

$$P(x,y) = \mathrm{circ}\left(\frac{\sqrt{x^2+y^2}}{w_{XP}}\right), \qquad (6)$$

- The coherent transfer function is

$$H(f_U, f_V) = \mathrm{circ}\left(\frac{\sqrt{(-\lambda z_{xp} f_U)^2 + (-\lambda z_{xp} f_V)^2}}{w_{XP}}\right), \qquad (7)$$

$$H(f_U, f_V) = \mathrm{circ}\left(\frac{\sqrt{f_U{}^2 + f_V{}^2}}{f_0}\right),$$

- where f_0 is again the coherent cutoff frequency as defined in Eq. (5).
- **<u>Unlike the square aperture, the cutoff frequency in this case is the same radially in all directions in the frequency plane.</u>**

Coherent Transfer Function Examples

- To observe or record a coherent image, the irradiance given by $I_i = |U_i|^2$ is actually measured.
- As a result of the squaring operation, the irradiance image can theoretically gain up to *twice* the frequency content of the field—think about the fact that

$$\cos^2(2\pi bx) = \tfrac{1}{2}\left[1 - \cos(2\pi 2bx)\right].$$

- **So, when an irradiance image is formed, the following cutoff should be considered:**

$$2f_0 = \frac{2w_{XP}}{\lambda z_{XP}}.$$

- Given that $2w_{XP} = D_{XP}$, and using $f/\# = \frac{z_{XP}}{D_{XP}}$

$$2f_0 = \frac{1}{\lambda(f/\#)}$$

Diffraction-limited Coherent Imaging Simulation

- One approach for simulating coherent imaging on the computer is based on Eq.(1) and implemented as:

$$U_i(u,v) = \Im^{-1}\{H(f_U, f_V)\Im\{U_g(u,v)\}\}. \quad (8)$$

- **<u>A simulation begins with a sampled ideal image, usually an image file</u>**.
- The sampled ideal image is assumed to have a physical sample interval Δu and side length L.

Diffraction-limited Coherent Imaging Simulation

- **The highest spatial frequency available in the ideal image is the Nyquist frequency $f_N = 1/(2\Delta u)$, so a diffraction-limited simulation requires**

$$2f_0 \le f_N . \qquad (9)$$

- This condition comes about because you can't model spatial frequencies in the simulation that are not present in the ideal image.
- **Applying Eq. (9), substituting for f_N and rearranging gives a criterion for the sample interval:**

$$\Delta u \le \frac{\lambda(f/\#)}{2} . \qquad (10)$$

Diffraction-limited Coherent Imaging Simulation

- Given $L = M\Delta u$, where M is the number of samples, then
$$L \le M \frac{\lambda(f/\#)}{2}. \tag{11}$$
- <u>**Consider a thin lens with a diameter of 12.5mm, a focal length of 125mm, and in which the wavelength of interest is 0.5μm.**</u>
- Assume a 250×250 sample array for an imaging simulation.
- First, calculate the *f*-number: $f/\# = \frac{125}{12.5} = 10.$
- Inserting the values in Eq. (10) yields
$$\Delta u \le \frac{0.5\mu\mathrm{m} \cdot 10}{2} = 2.5\,\mu\mathrm{m},$$

Diffraction-limited Coherent Imaging Simulation

- The side length constraint is:

$$L \leq 250 \times 2.5\ \mu\text{m} = 0.625\ \text{mm}. \tag{12}$$

- The image plane size is limited to 0.625×0.625 mm (small area).
- Working with a larger array increases the image size.
- For example, with a 2048×2048 sample array, the maximum image plane size would be 5.12×5.12mm—still a relatively small area.

Diffraction-limited Coherent Imaging Simulation

- This illustrates that the Fourier optics-based simulation examines a small part of the image plane for near diffraction-limited performance.
- **However, a very large array is needed to model a modest field of view, which might correspond to an image size of, say, 10 or 20 mm in this case.**

Diffraction-limited Coherent Imaging Simulation

- Let's work up some code: we use a 250 × 250 pixel image file that is a USAF 1951 resolution test chart.
- The USAF 1951 is still used today for testing lenses and optical systems.
- Enter the following:

```
%  coh_image Coherent Imaging Example

A=imread('USAF1951B250','png'); %read image file
[M,N]=size(A);               %get image sample size
A=flipud(A);                 %reverse row order
Ig=single(A);                %integer to floating
Ig=Ig/max(max(Ig));          %normalize ideal image

ug=sqrt(Ig);                 %ideal image field
L=0.3e-3;                    %image plane side length (m)
du=L/M;                      %sample interval (m)
u=-L/2:du:L/2-du; v=u;

figure(1)                    %check ideal image
imagesc(u,v,Ig);
colormap('gray');  xlabel('u (m)'); ylabel('v (m)');
axis square
axis xy
```

Diffraction-limited Coherent Imaging Simulation

- Since the image file represents an irradiance image, take the square root to get the magnitude of the field (zero phase is assumed in the ideal image).
- The side length is L = 0.3 mm, which satisfies Eq. (12), and with M = 250 the sample interval is Δu = 1.2×10^{-6} m.

Diffraction-limited Coherent Imaging Simulation

- <u>**The ideal test chart image is shown in Fig. 2.**</u>

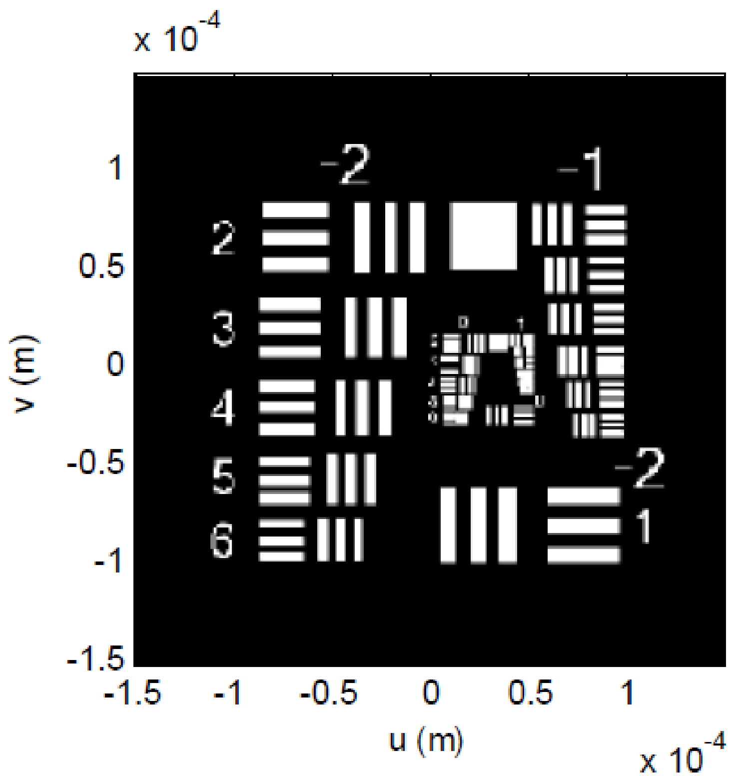


Fig. 2. USAF test chart ideal image.

Diffraction-limited Coherent Imaging Simulation

- Add the following code to define the imaging system parameters and generate the coherent transfer function:

```
lambda=0.5*10^-6;        %wavelength
wxp=6.25e-3;             %exit pupil radius
zxp=125e-3;              %exit pupil distance
f0=wxp/(lambda*zxp);     %cutoff frequency

fu=-1/(2*du):1/L:1/(2*du)-(1/L); %freq coords
fv=fu;
[Fu,Fv]=meshgrid(fu,fv);
H=circ(sqrt(Fu.^2+Fv.^2)/f0);

figure(2)                %check H
surf(fu,fv,H.*.99)
camlight left; lighting phong
colormap('gray')
shading interp
ylabel('fu (cyc/m)'); xlabel('fv (cyc/m)');
```

Diffraction-limited Coherent Imaging Simulation

- Visible wavelength illumination is assumed.
- For the f /10 lens the following are computed:

$$f_N = 1/(2 \cdot 1.2 \times 10^{-6}) = 4.17 \times 10^5 \text{ cycles/m},$$

$$2f_0 = 1/(0.5 \times 10^{-6} \cdot 10) = 2 \times 10^5 \text{ cycles/m},$$

$$f_0 = 1 \times 10^5 \text{ cycles/m}.$$

- Thus, Eq. (9) is satisfied.
- The coherent transfer function is shown in Fig. 3.

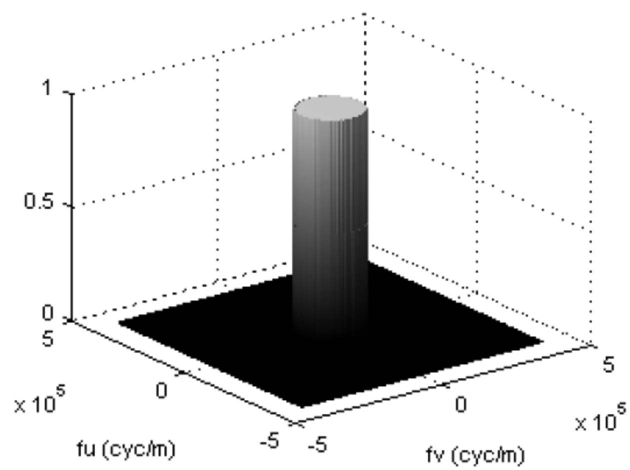


Fig. 3. Coherent transfer function.

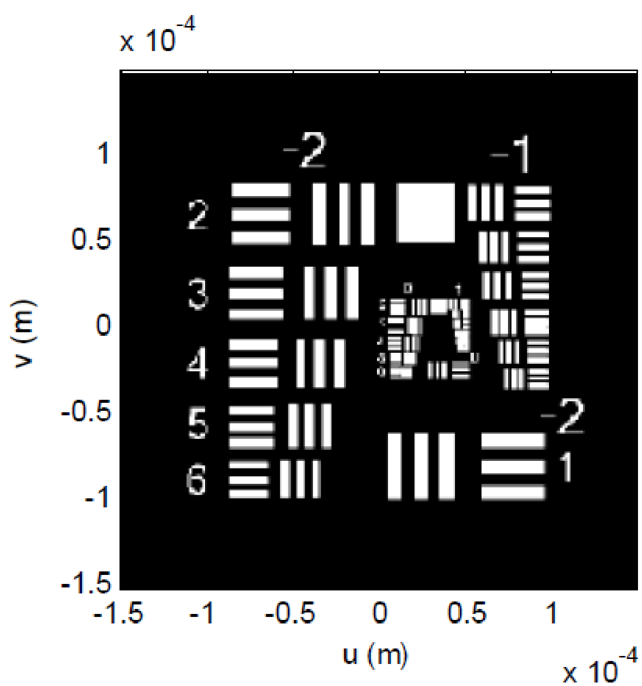
Diffraction-limited Coherent Imaging Simulation

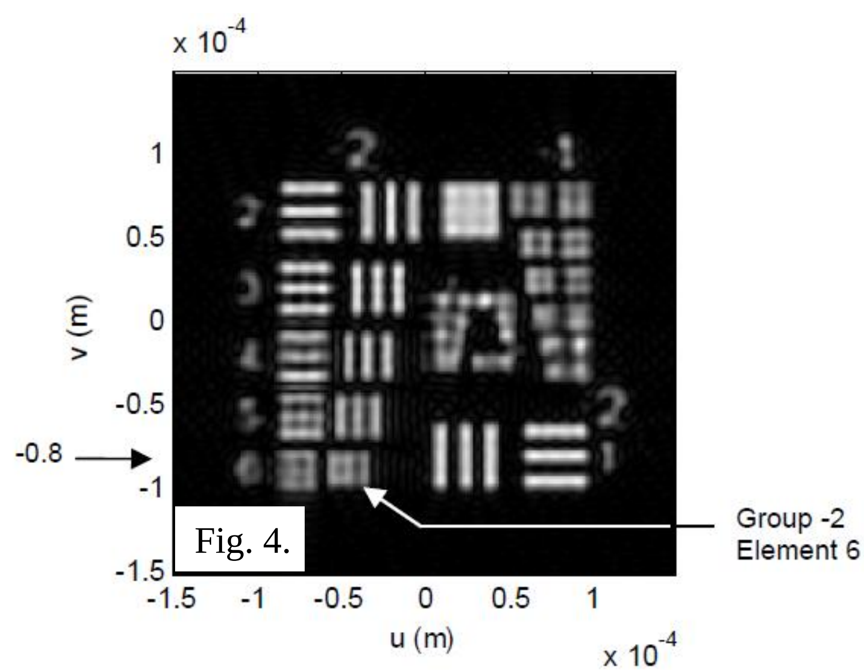
- **If Eq. (9) were violated, the pupil function in Fig. 3 would reach beyond halfway to the array boundaries.**
- Add the following to generate the simulated image:

```
H=fftshift(H);
Gg=fft2(fftshift(ug));
Gi=Gg.*H;
ui=ifftshift(ifft2(Gi));
Ii=(abs(ui)).^2;
figure(3)                %image result
imagesc(u,v,nthroot(Ii,2));
colormap('gray'); xlabel('u (m)'); ylabel('v (m)');
axis square
axis xy
figure(4)                %horizontal image slice
vvalue=-0.8e-4;         %select row (y value)
vindex=round(vvalue/du+(M/2+1)); %convert row index
plot(u,Ii(vindex,:),u,Ig(vindex,:),':');
xlabel('u (m)'); ylabel('Irradiance');
```

Diffraction-limited Coherent Imaging Simulation

- **<u>Diffraction-limited coherent imaging is implemented and the resulting image is displayed in Fig. 4.</u>**
- The features are blurred and some of the three-bar groups are unresolved.





Simulated diffraction-limited coherent image

Diffraction-limited Coherent Imaging Simulation

- In Fig. 4, the three bars of Group −2, Element 6, appear to be the smallest group that is “resolved.”
- Selecting the v-coordinate of -0.8×10^{-4}m in the profile code gives the display in Fig. 5.
- The large bars of Group −2, Element 1 are clearly resolved but with some obvious ringing effects.
- Element 6 vertical bars are resolved but with much less contrast.

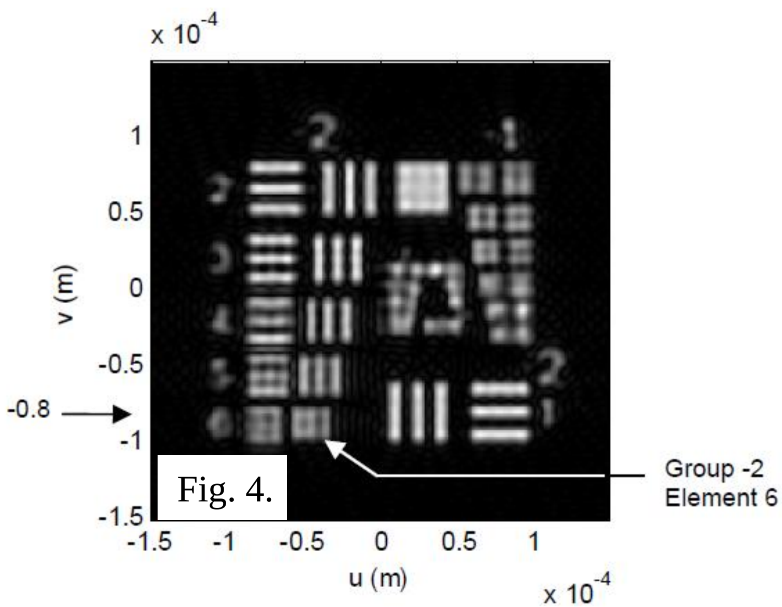


Fig. 4.

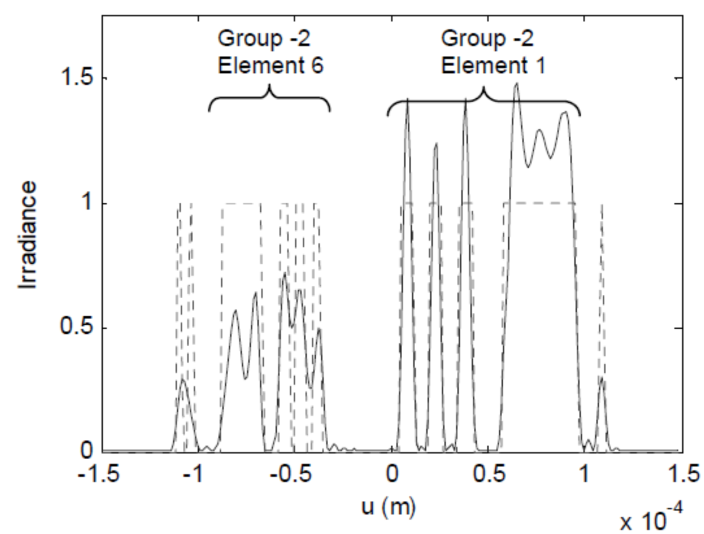


Fig. 5.

Diffraction-limited Coherent Imaging Simulation

- **Figure 6 shows a sequence of image spectra with the coherent transfer function and resulting irradiance images for different pupil sizes (*f*-numbers).**

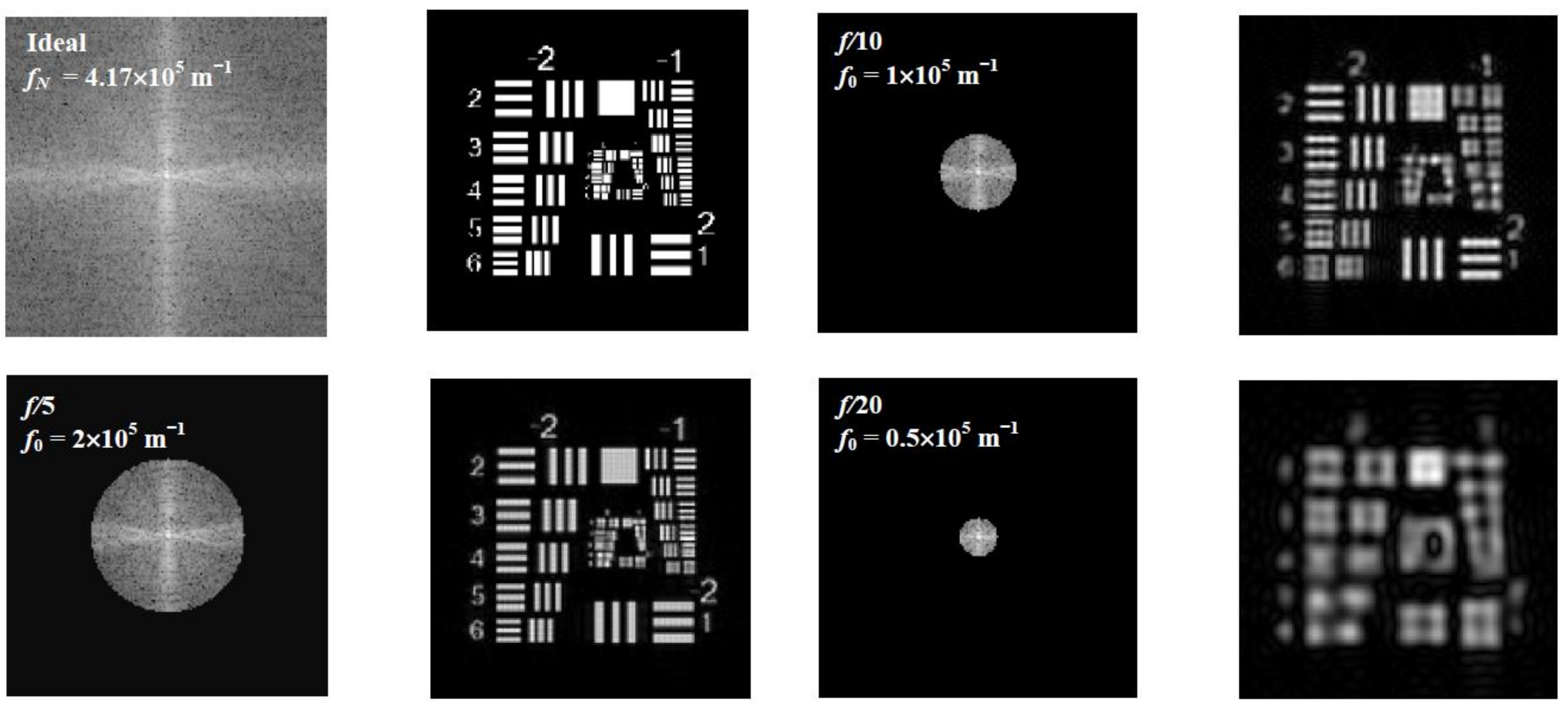


Fig. 6. Coherent image spectral magnitude (left column, log scaled) and associated irradiance images (right column).