Optical Signal Processing

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Chapter 3
Propagation Simulation

Fresnel Transfer Function (TF) Propagator

- The Fresnel diffraction expression is often the approach of choice for simulations since **it applies to a wide range of propagation scenarios and is relatively straightforward to compute**.
- A common propagation routine is based on the following equation:

$$U_2(x,y) = \Im^{-1}\left\{\Im\{U_1(x,y)\}H(f_X, f_Y)\right\},$$

- The transfer function *H is* given by

$$H(f_X, f_Y) = e^{jkz}\exp\left[-j\pi\lambda z\left(f_X^2 + f_Y^2\right)\right].$$

- This propagator function takes the source field U_1 and produces the observation field U_2 where the source and observation side lengths and sample coordinates are identical.

Fresnel Transfer Function (TF) Propagator

- Start a New M-file and save it with name “propTF.”
- Enter the following function:

```
function[u2]=propTF(u1,L,lambda,z);
% propagation - transfer function approach
% assumes same x and y side lengths and
% uniform sampling
% u1 - source plane field
% L - source and observation plane side length
% lambda - wavelength
% z - propagation distance
% u2 - observation plane field

[M,N]=size(u1);                 %get input field array size
dx=L/M;                         %sample interval
k=2*pi/lambda;                  %wavenumber

fx=-1/(2*dx):1/L:1/(2*dx)-1/L; %freq coords
[FX,FY]=meshgrid(fx,fx);

H=exp(-j*pi*lambda*z*(FX.^2+FY.^2));  %trans func
H=fftshift(H);                  %shift trans func
U1=fft2(fftshift(u1));          %shift, fft src field
U2=H.*U1;                       %multiply
u2=ifftshift(ifft2(U2));        %inv fft, center obs field
end
```

Fresnel Transfer Function (TF) Propagator

- A propagation approach can be devised based on the following equation:

$$U_2(x,y) = \Im^{-1}\{\Im\{U_1(x,y)\}\Im\{h(x,y)\}\}.$$

- The impulse response *h* is given by,

$$h(x,y) = \frac{e^{jkz}}{j\lambda z}\exp\left[\frac{jk}{2z}\left(x^2+y^2\right)\right].$$

- **Even though the two Eqs. represent identical analytical operations, with discrete sampling and transforms, the transfer function and impulse response approaches can yield different results**.

Fresnel Transfer Function (TF) Propagator

```
function[u2]=propIR(u1,L,lambda,z);
% propagation - impulse response approach
% assumes same x and y side lengths and
% uniform sampling
% u1 - source plane field
% L - source and observation plane side length
% lambda - wavelength
% z - propagation distance
% u2 - observation plane field

[M,N]=size(u1);              %get input field array size
dx=L/M;                      %sample interval
k=2*pi/lambda;               %wavenumber

x=-L/2:dx:L/2-dx;            %spatial coords
[X,Y]=meshgrid(x,x);

h=1/(j*lambda*z)*exp(j*k/(2*z)*(X.^2+Y.^2)); %impulse
H=fft2(fftshift(h))*dx^2; %create trans func
U1=fft2(fftshift(u1));    %shift, fft src field
U2=H.*U1;                 %multiply
u2=ifftshift(ifft2(U2));  %inv fft, center obs field
end
```

Sampling Regimes and Criteria

- **Both conditions are only satisfied when we use the critical sampling condition**.

$$\Delta x = \frac{\lambda z}{L}$$

- This is the critical sampling situation where the sampled *H* and *h* functions as an FFT pair, **turn out to have values that exactly match the analytic functions *H* and *h***.
- Under this condition the full bandwidth of the sampled array ($1/2\Delta x$) is available for modeling the source, and the full area of the array in the observation plane can be used.

Fraunhofer Propagation

- **<u>The expression for the Fraunhofer pattern is repeated here:</u>**

$$U_2(x_2, y_2) = \frac{\exp(jkz)}{j\lambda z}\exp\left[j\frac{k}{2z}\left(x_2^2 + y_2^2\right)\right]$$

$$\times \iint U_1(x_1, y_1)\exp\left[-j\frac{2\pi}{\lambda z}(x_2 x_1 + y_2 y_1)\right]dx_1 dy_1,$$

- where, for coding purposes, the source plane variables are indicated with the subscript 1 and the observation plane variables with subscript 2.
- **<u>When using the FFT to compute the Fraunhofer field, the source and observation plane side lengths are not generally the same</u>**.

Fraunhofer Propagation

- From Eq. (11-1) in Chap. 2,

$$\lambda z f_{X1} \rightarrow x_2 ,$$

- Using Eq. (6-1) in Chap. 1, the observation plane side length and sample interval are found in terms of the source plane parameters

$$L_2 = \frac{\lambda z}{\Delta x_1}, \quad \text{and} \quad \Delta x_2 = \frac{\lambda z}{L_1}. \qquad (14)$$

- **<u>The observation plane coordinates are given as</u>**

$$x_2 = \left[\frac{-L_2}{2} : \Delta x_2 : \frac{L_2}{2} - \Delta x_2\right] = \lambda z \left[\frac{-1}{2\Delta x_1} : \frac{1}{L_1} : \frac{1}{2\Delta x_1} - \frac{1}{L_1}\right].$$

Fraunhofer Propagation

- If critical sampling is used ($\Delta x_1 = \lambda z / L_1$), then Eq. (14) indicates that the side lengths will be equal $L_2 = L_1$.
- Otherwise, the side lengths are different.
- **The function propFF that computes the Fraunhofer pattern follows**:

```
function[u2,L2]=propFF(u1,L1,lambda,z);
% propagation - Fraunhofer pattern
% assumes uniform sampling
% u1 - source plane field
% L1 - source plane side length
% lambda - wavelength
% z - propagation distance
% L2 - observation plane side length
% u2 - observation plane field
%
```

Fraunhofer Propagation

```
[M,N]=size(u1);                 %get input field array size
dx1=L1/M;                       %source sample interval
k=2*pi/lambda;                  %wavenumber
%
L2=lambda*z/dx1;                %obs sidelength
dx2=lambda*z/L1;                %obs sample interval
x2=-L2/2:dx2:L2/2-dx2;          %obs coords
[X2,Y2]=meshgrid(x2,x2);
%
c=1/(j*lambda*z)*exp(j*k/(2*z)*(X2.^2+Y2.^2));
u2=c.*ifftshift(fft2(fftshift(u1)))*dx1^2;
end
```

Fraunhofer Propagation

- To use this, make the following changes in the “sqr_beam” matlab code which are described in the next page:

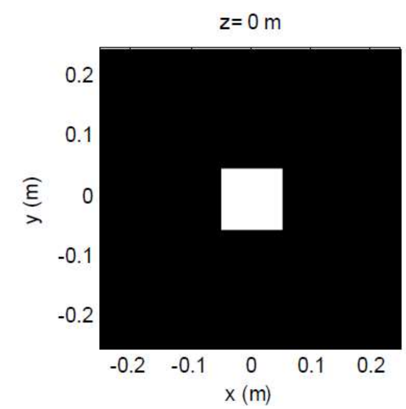
```
w=0.011;                    %source half width (m)

[u2,L2]=propFF(u1,L1,lambda,z);

dx2=L2/M;
x2=-L2/2:dx2:L2/2-dx2; %obs  ords
y2=x2;
I2=abs(u2.^2);          %obs irrad
imagesc(x2,y2,nthroot(I2,3));%stretch image contrast
```

Square Beam Example

- Now it is time to try out the TF or IR propagators.
- Consider a source plane with dimensions 0.5m × 0.5m ($L1$ = 0.5m).
- Start a New M-file and use the name "sqr_beam."
- Enter the following:




```
% sqr_beam propagation example
%
L1=0.5;                 %side length
M=250;                  %number of samples
dx1=L1/M;               %src sample interval
x1=-L1/2:dx1:L1/2-dx1; %src coords
y1=x1;
```

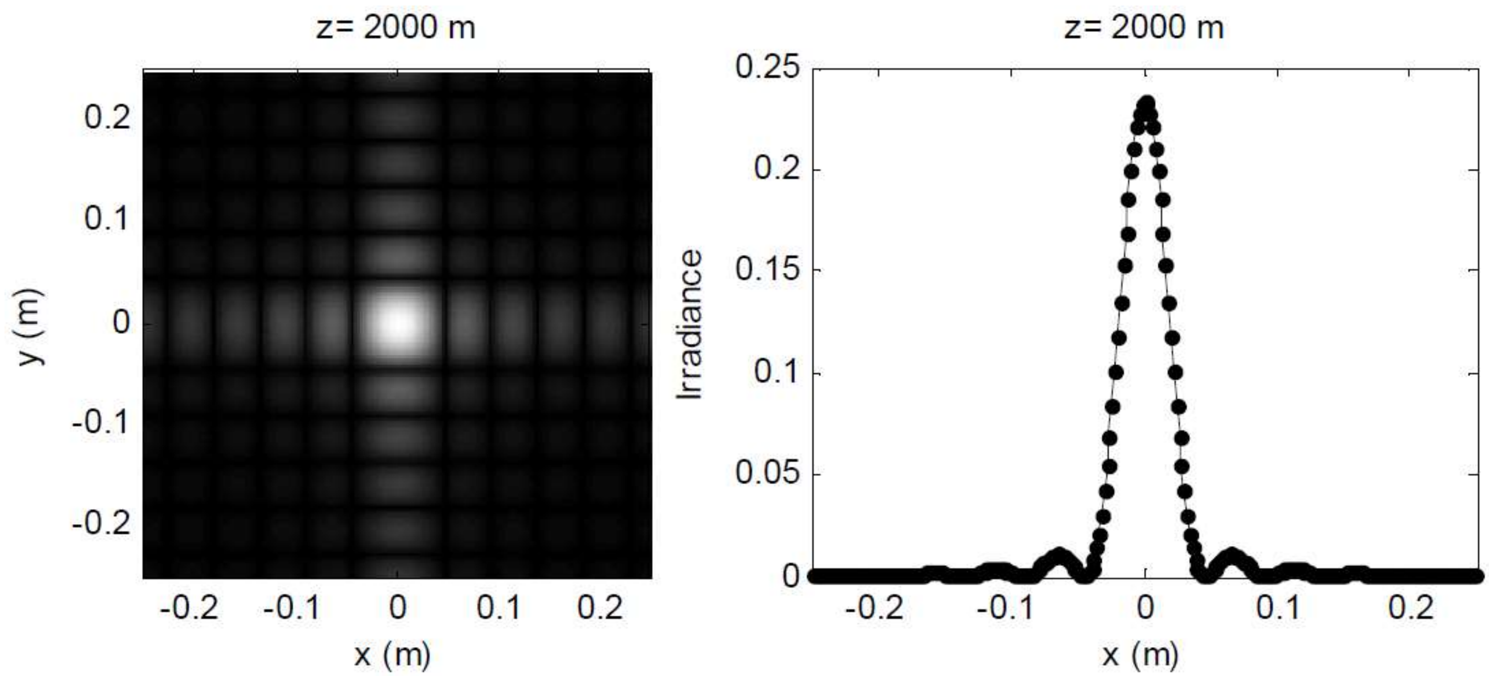
Square Beam Example

```
lambda=0.5*10^-6;      %wavelength
k=2*pi/lambda;         %wavenumber
w=0.051;               %source half width (m)
z=2000;                %propagation dist (m)

[X1,Y1]=meshgrid(x1,y1);
u1=rect(X1/(2*w)).*rect(Y1/(2*w)); %src field
I1=abs(u1.^2);         %src irradiance
%
figure(1)
imagesc(x1,y1,I1);
axis square; axis xy;
colormap('gray'); xlabel('x (m)'); ylabel('y (m)');
title('z= 0 m');
```

Fraunhofer Propagation

- **The simulation result can be checked against the analytic Fraunhofer result.**



(a) (b)

Fraunhofer Propagation

- Take the Fourier transform of the source distribution:

$$\Im\left\{\text{rect}\left(\frac{x_1}{2w}\right)\text{rect}\left(\frac{y_1}{2w}\right)\right\} = 4w^2 \,\text{sinc}(2wf_{X1})\text{sinc}(2wf_{Y1}).$$

- Substitute $x_2/\lambda z$ for f_{X1} and $y_2/\lambda z$ for f_{Y1} and include the multipliers to get the Fraunhofer field:

$$U_2(x_2, y_2) = \frac{\exp(jkz)}{j\lambda z}\exp\left[j\frac{k}{2z}\left(x_2^{\,2} + y_2^{\,2}\right)\right]$$

$$\times 4w^2 \,\text{sinc}\left(\frac{2w}{\lambda z}x_2\right)\text{sinc}\left(\frac{2w}{\lambda z}y_2\right).$$

Fraunhofer Propagation

- The irradiance pattern is $I_2(x_2, y_2) = |U_2(x_2, y_2)|^2$, which yield

$$I_2(x_2, y_2) = \left(\frac{4w^2}{\lambda z}\right)^2 \mathrm{sinc}^2\left(\frac{2w}{\lambda z}x_2\right)\mathrm{sinc}^2\left(\frac{2w}{\lambda z}y_2\right).$$

- Suppose the Fraunhofer field is of interest with the chirp term, $\exp[jk(2z)^{-1}(x_2^2 + y_2^2)]$.
- **The chirp function will be adequately sampled in the observation plane if $\Delta x_2 \leq \lambda z / L_2$, or equivalently, by applying Eq. (14) when the source plane sampling is**

$$\Delta x_1 \geq \frac{\lambda z}{L_1}. \qquad (15)$$

- If Eq.(15) is not satisfied, the chirp phase is aliased.